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SPECTROSCOPIC DETERMINATION OF SOLAR PARALLAX.

By ARTHUR B. TURNER.

I.

The average velocity of the Earth around the Sun is about nineteen miles per second. The velocity of a star, in the ecliptic and in quadrature with the Sun, as determined spectroscopically, will be affected by this whole amount, and the difference of the velocities at two successive quadratures will give twice the velocity of the Earth. Whence easily follows the Earth's distance from the Sun and the solar parallax. The sum of the two velocities will give twice the velocity of the star with reference to the Sun. In the diagram (Earth's orbit a circle and star in the ecliptic), the Earth at E is moving directly towards the star. If S is the velocity at which the two bodies are coming together as given by the spectroscope, then,

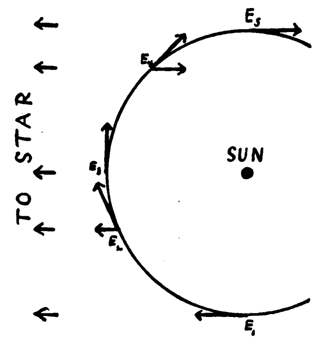
$$(1)\quad V_* + V_E = S, \text{ where } \begin{cases} V_* = \text{velocity of star} \\ V_E = \text{velocity of Earth} \end{cases}$$

At the next quadrature, E_5, the Earth is moving away from the star and

$$(2)\quad V_* - V_E = S',$$

S′ being the spectroscopic velocity of separation of the two bodies. Solving these two equations, we have,

$$V_E = \frac{S - S'}{2} \text{ and } V_* = \frac{S + S'}{2}$$



The velocity of the Earth in its circular orbit is given by the equation,

$$V_E = \frac{2\pi a}{T}, \text{ where } \begin{cases} a = \text{radius of orbit} \\ T = \text{Earth's period} \end{cases}$$

$$\text{or } a = \frac{TV_E}{2\pi}, \text{ and}$$

$$\pi \text{ (solar parallax)} = 206{,}265'' \frac{R}{a}$$

where R = equatorial radius of the Earth. Differentiating and substituting, we find,

$$d\pi = -\frac{\pi}{V_E} d V_E, \text{ or } d V_E = -V_E \frac{d\pi}{\pi}$$

indicating the rate of change of the parallax (π) with the velocity.

As an example, suppose,

$$\pi = 8''.80,\ V_E = 19 \text{ miles, and } d V_E = \pm 0.1 \text{ mile,}$$
$$\text{then } d\pi = \mp 0''.046$$

That is, an error of ± 0.1 mile in the determination of the Earth's velocity causes an error of $\mp 0''.046$ in the value of the parallax determined from it.

II.

This apparently simple solution of the problem of finding the Sun's distance is a rather rough approximation, for the ideal conditions as outlined above do not exist in nature. In the first place, stars of a suitable spectral type and magnitude

for velocity determinations are not found exactly in the ecliptic; second, the Earth's orbit is slightly elliptical, which makes the Earth's velocity a variable quantity; third, it also rotates on an axis and the observer has a component of velocity relative to the star depending upon the hour angle of the observation; fourth, the Earth and Moon revolve about a common center of gravity, and planetary perturbations also change slightly the elliptic velocity of the Earth; fifth, some of the larger planets, like *Jupiter* and *Saturn,* swing the Sun out of position sufficiently to affect the star's motion with respect to the Sun's center; sixth, the star may be a spectroscopic binary and have a variable velocity with respect to the observer (the component of the Sun's motion through space in the direction of a star is assumed to be constant); and, seventh, the observations cannot be taken exactly at quadrature with the Sun for a number of plates, as spectrograms must be taken of each star at successive quadratures.

These departures from ideal conditions can be allowed for by the computation of corrections, which when applied will give a resulting value of the solar parallax having a great degree of accuracy.

III.

I shall now review briefly a piece of work started by Sir David Gill at the Royal Observatory, Cape of Good Hope, and completed under Director S. S. Hough, published in the *Annals* of the Cape Observatory, Vol. X, Part III, (1909). This is not the first application of the spectroscopic method of determining the solar parallax, but it is the latest and most accurate in its results. Professor Küstner of Bonn first announced, in 1905,[1] a value of $\pi = 8''.829 \pm 0''.013$ from seventeen spectrograms of α *Boötis* taken near two successive quadratures.

From February, 1906, to May, 1908, 302 spectrograms were taken with the Victorian telescope, distributed among the following seven stars:—

α *Tauri*	25 plates
α *Orionis*	35 "

[1] *Astronomische Nachrichten,* 4048-49.

α *Canis Minoris*	61 plates
β *Geminorum*	22 "
α *Boötis*	55 "
α_2 *Centauri*	55 "
α *Scorpii*	49 "

These cover twelve hours of right ascension from $4^h\ 30^m$ to $16^h\ 23^m$ (see diagram), and range in latitude from $-45°$ to $+30°$, the nearest star being $5°$ from the ecliptic. Five of the stars are south of the ecliptic. The table below shows the intervals near quadratures for the plates of α *Scorpii*:—

I.	13 plates	Feb. 21-Mar. 23, 1906
II.	12 "	Aug. 13-Sept. 24, "
III.	8 "	Mar. 7-Mar. 11, 1907
IV.	12 "	Aug. 12-Sept. 6, "
V.	4 "	Mar. 9-Mar. 16, "

To get the velocity from these, five plates were taken of the Sun and each compared with one plate of the star (selected at random), called a "standard plate," by the Hartmann spectro-comparator.[1] This is an optical arrangement by which two spectra are brought into the same field of view and the shift of one plate relatively to the other, required to make portions of corresponding lines coincide, can be measured by a micrometer. The mean of the five comparisons, corrected for annual and diurnal motion of the observer with reference to the Sun, gave the shift of the lines of the "standard plate" relatively to the normal Sun. In all the comparisons the same twelve iron lines were used, ranging in wave-length from $\lambda = 4427.482$ to $\lambda\,4603.126$ (ROWLAND).

Then each star plate was compared with the standard of that star, using the same twelve lines. To the shift thus measured was added that of the standard, and the corrections to the Sun, calculated on the assumption that $\pi = 8''.800$. The resultant is the radial velocity of the star. The correction due to the velocity of the Earth was computed by Dr. SCHLESINGER'S formula,[2] which includes the small lunar and planetary terms necessary to the accuracy of the present problem.

[1] *Astrophysical Journal*, Vol. XXIV.
[2] *Astrophysical Journal*, Vol. IX.

Imagine three rectangular axes passed through the Sun's center and let the Earth at any instant be referred to them, then the changes in the co-ordinates in a unit of time will represent the velocities of the Earth parallel to the axes. The sum of the resolved components in the direction of the star of each of these velocities (represented by ΔX, ΔY, ΔZ) will give the component of the Earth's velocity with reference to the star. ΔX, ΔY, ΔZ were taken from the Berlin *Jahrbuch*, $\pi = 8''.800$, and BESSEL'S equatorial radius of the Earth ($6{,}377.397^{km}$) were used in reducing the velocity to kilometers per second.

IV.

The means of the adopted corrections for the Sun ($\odot$) and the means of the observed velocities for each quadrature of a star gave an equation of the form:

$$\text{Observed velocity} = V + \odot \frac{d\,\pi}{\pi} + \mu f\,(t - t_0)$$

Three or more such equations were obtained for each star and solved (by least squares) for $\frac{d\,\pi}{\pi}$ and the velocity (V) of the star for the epoch t_0. The term $\mu f\,(t - t_0)$ was introduced to take care of the variability of the star's velocity (spectroscopic binary), t_0 being the epoch of the apex of the velocity curve and μ a constant. By making several assumptions with respect to the function f, the interesting fact developed that the value of π was practically independent of this term; that is, whether the velocity of the star was constant, proportional to the time (t), or to the square of the time (t^2). The following results were obtained under the assumption that the radial velocities of all the stars change uniformly with the time:—

Star	π
α *Tauri*	8''.832
α *Orionis*	8 .835
α *Canis Minoris*	8 .763
β *Geminorum*	8 .783
α *Boötis*	8 .807
α_2 *Centauri*	8 .778
α *Scorpii*	8 .805

Mean result, $\pi = 8''.799 \pm 0''.0068$

The final most probable value of the parallax was $\pi = 8''.800 \pm 0''.006$, which agrees remarkably well with the value announced by A. R. HINKS[1] from a discussion of some hundreds of plates of *Eros* taken at twelve observatories during the opposition of 1900. He gives $\pi = 8''.807 \pm 0''.0027$. This amounts to knowing the Sun's mean distance within about 30,000 miles.

V.

The observing programme of 365 stars now under way at the Cape Observatory will contain when completed the radial velocities of at least fifty stars, observed near quadrature with the Sun, suitable for the determination of the solar parallax. So that in a few more years the spectroscopic determination of this important astronomical constant—the scale of the solar system—may equal in accuracy the aberration method (rated by Professor YOUNG among the best), to which it is closely related, depending, as it does, on the ratio of the velocity of the Earth to the velocity of light.

COLLEGE OF CITY OF NEW YORK,
August 24, 1912.

ORBITS OF THE VISUAL AND SPECTROGRAPHIC BINARY STAR *EPSILON HYDRÆ* AB.[2]

BY R. G. AITKEN.

It has not been possible up to the present time to verify the orbit elements of a visual binary star by independent computations based upon measures of the radial velocities in the system, and it appears that the number of cases in which such verification will be practicable will always be small. As a rule, the revolution periods of the visual binaries are so great that the orbital velocities of the components, except at periastron, are very small. The low inclination of the orbit planes of some systems, the ill-defined lines in the spectra of others and in

[1] *Monthly Notices*, February, 1912.

[2] Read at the meeting of the Astronomical and Astrophysical Society of America, Allegheny, Pa., August 27-30, 1912.